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Invention of the Plane Geometrical Formulae - Part III

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Abstract: In this paper , I have invented the first new type of a scalene quadrilateral and also developed the two new formulae for finding the area of that's new type of quadrilateral. My finding are based on phthagoras theorem.

I. Introduction

The types of quadrilaterals named on the basis of their angles and side are square, rectangle, rhombus, parallelogram, trapezium and kite respectively. These various types of quadrilaterals are also exist in educational curriculum.

I Have invented the first new type of a scalene quadrilateral and also developed the two new formulae for finding the area of it's scalene quadrilateral with the help of Pythagoras theorem.I Used Pythagoras theorem with geometrical figures and algebraic equations for the invention of the new type of a quadrilateral.

I Proved it by using geometrical formulae & figures, 10 examples & verifications (Proofs).
Here, myself is giving you the summary of the plane geometrical formulae-Part-III

<u>First new type of a quadrilateral is as follows:-</u>

1) Scalene quadrilateral :-

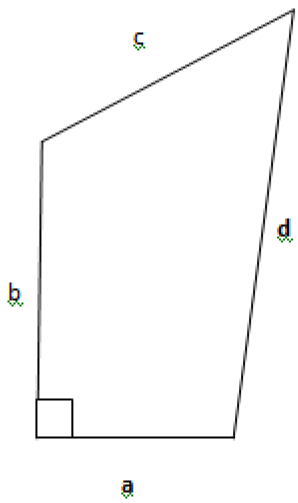

Fig No. – (1)

If the four sides of a quadrilateral are of different lengths. The lengths of all four sides of that quadrilateral are a, b, c and d respectively. The sides a,b, c and d are positive integers. The lengths of the first three consecutive sides(a, b, c) is a set of Pythagorean Triple. In Pythagorean Triple, a is the length of smallest side.

In this new type of quadrilateral , a is the length of smallest side also and the length fourth side (d) is twice the length of smallest side(d) . The smallest side and the fourth side are also adjacent sides.

Similarly, a and b are the lengths of both sides forming the right angle. It means that one angle of that's quadrilateral is a right angle means measure 90^0. It's diagonals are not congruent.

A quadrilateral with all four sides of different lengths and one angle a right angle, then this quadrilateral is named as a scalene quadrilateral. Hence, this scalene quadrilateral is the first new type of a quadrilateral.

II. Construction:-

Construct $\square$ PQRS such that
l(QR) = 3cm , l(PQ) = 4 cm
l(PS) = 5 cm , l(RS) = 6 cm
and m $\angle$ PQR = 90^0

III. Steps of construction:-

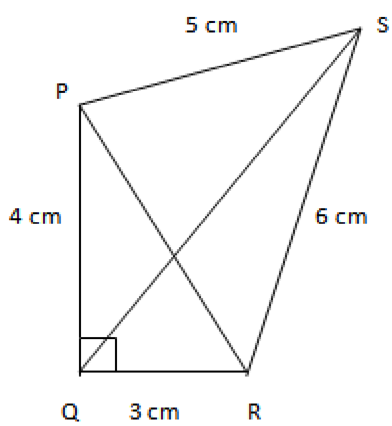

Fig No.- (2)

1) Seg QR is drawn measuring 3 cm .
2) At Point Q of seg QR , a ray is drawn making an angle of 90^0 with it, as shown.
3) On this ray, point P lies at a distance of 4 cm from point Q . Therefore, with point Q as the centre and radius of 4 cm , an arc is drawn to intersect the ray .The point of intersection is labeled as P.
4) Drawn an arc on one side of seg PQ with center P and radius 5 cm.
5) Drawn another arc with center R and radius 6 cm to cut the first arc.

The Point of intersection is labeled as S.

6) Thus, the positions of all four vertices of □PQRS are fixed.
7) Seg PS and seg RS are drawn and the required PQRS is obtained.
8) Draw seg PR and seg QS.

IV. Explanation :-

In □ PQRS ,
L(QR) = 3 cm , *l*(PQ) = 4 cm
l(PS) = 5 cm , *l*(RS) = 6 cm
and m ∠PQR = 90^0
In above required □PQRS ,

The four sides of a quadrilateral are of different lengths. The lengths of all four side of PQRS are 3 cm, 4 cm, 5 cm and 6 cm respectively. The lengths of the first three consecutive sides (3 cm,4 cm, 5 cm) is a set of Pythagorean Triple . In Pythagorean Triple ,3 cm is the length of smallest side.

In □ PQRS , 3 cm is the length of smallest side QR also and the length of fourth side RS (6cm) is double the length of smallest side QR (3cm).

Similarly , side PQ and side QR are both sides forming the right angle. It means that, one angle, PQR of PQRS is a right angle means measure 90^0 .Seg PR and seg QS are the diagonals of required PQRS.

□PQRS with all four sides of different lengths and one angle a right angle, then this quadrilateral is a scalene quadrilateral.

Hence,□ PQRS is the first new type of a scalene quadrilateral.

Properties of a scalene quadrilateral

1) Four sides of a scalene quadrilateral are of different lengths.
2) Out of four angles , one angle is a right angle means measure of 90^0.
3) The diagonals of a scalene quadrilateral are not congruent.
4) The diagonals of a scalene quadrilateral intersect each other. But they do not have perpendicular bisector with each other.
5) The diagonals of a scalene quadrilateral do not bisect each other.

Now using this above constru□ion of PQRS We shall obtain the new formulae for finding the area of a scalene quadrilateral (first new type).

Method :-

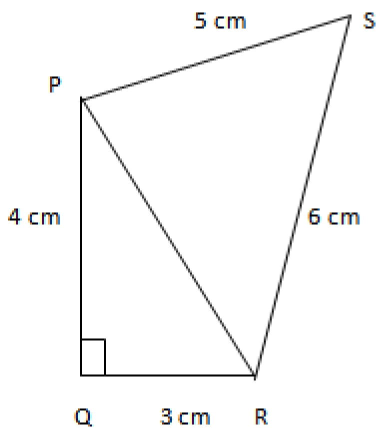

Fig No. – (3)

In □ PQRS , the diagonal PR is drawn. Due to the diagonal PR, □ PQRS is divided into two triangles, namely,△PQR and △ PRS.

From □ PQRS,
1) Taking First △PQR.

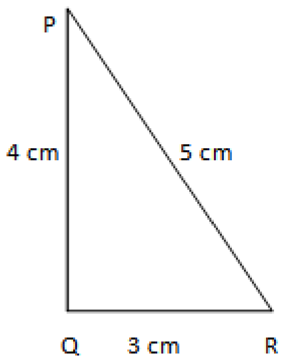

Fig No. – (4)

In △ PQR, Seg PQ and seg QR are both sides forming the right angle. Seg PR is it's hypotenuse.
L(QR) = 3 cm, *l*(PQ) = 4 cm And m ∠ PQR = 90^0
(3 cm, 4 cm, 5 cm) is a Pythagorean Triple.
Therefore , the length of hypotenuse is 5 cm.
L(PR) = 5 cm.

2) Let us now taking △PRS.

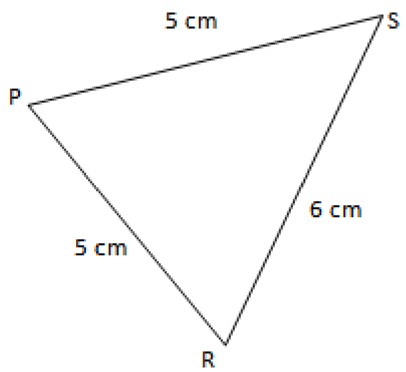

Fig No. – (5)

In △ PRS,

The length of seg PS and the length of seg PR are equal.

L(PS) = *l*(PR) = 5 cm.

Hence, △ PRS is an isosceles triangle.

3) Let us now taking an isosceles triangle △ PRS.

Similarly,

In an isosceles △ triangle PRS, perpendicular PM is drawn on side RS from the vertex P.

By the length of perpendicular PM drawn on the side RS from the vertex P, two congruent right angled triangle are formed. These two congruent right angled triangle are namely PMR and PMS.

Due to the perpendicular PM drawn on side RS , side RS is divided into two equal segments. Therefore these two equal segments are seg MR and seg MS.

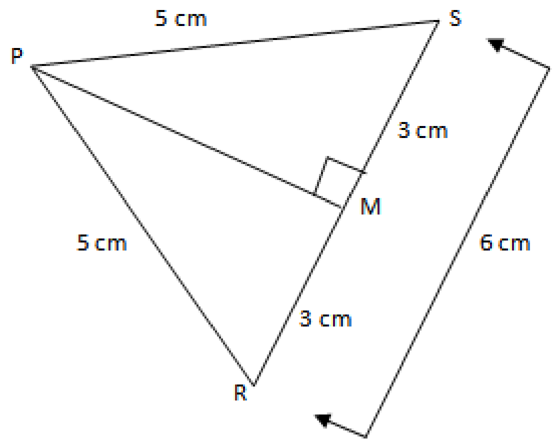

Fig No. – (6)

L(MR) = *l*(MS) = ½x *l*(RS)

l (MR) = *l*(MS) = ½ x 6

l(MR) = *l*(MS) = 3 cm.

But the length of side QR of □ PQRS is 3 cm

l(QR) = 3 cm

l(MR) = *l*(MS) = *l*(QR) = 3 cm

4) Now taking a right angled triangle △ PMR.

In △ PMR, Seg PM and seg MR are both sides forming the right angle. Seg PR is it's hypotenuse.

*l(*MR) = 3 cm, ∠Hypotenuse , *l(*PR) = 5 cm and m PMR = 90^0

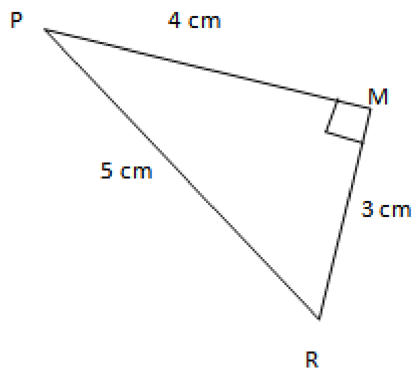

Fig No. – (7)

BY Using Pythagoras Theorem,

$PR^2 = PM^2 + MR^2$ △ △

$PM^2 + MR^2 = PR^2$

$PM^2 + 3^2 = 5^2$

$PM^2 + 9 = 25$

$PM^2 = 16$

$PM^2 = 4^2$

Removing the square on both side

PM = 4 cm

In an isosceles △ PRS,

Seg PM is perpendicular to side RS. Therefore , PM is the height.

The length of perpendicular, PM and the length of side PQ of □ PQRS are equal.

L(PM) = 4 cm

From above information, formed two right angled triangles, namely, △ PMR and △ PMS are congruent △ to another right angled triangle, △ PQR.

It means that, three congruent right angled triangles, namely ,

PMR, PMS and PQR are also formed in PQRS.

m PQR = ∠m PMR ∠= m PM ∠S = 90^0

Generalization of a scalene quadrilateral (□ PQRS)
Method : -

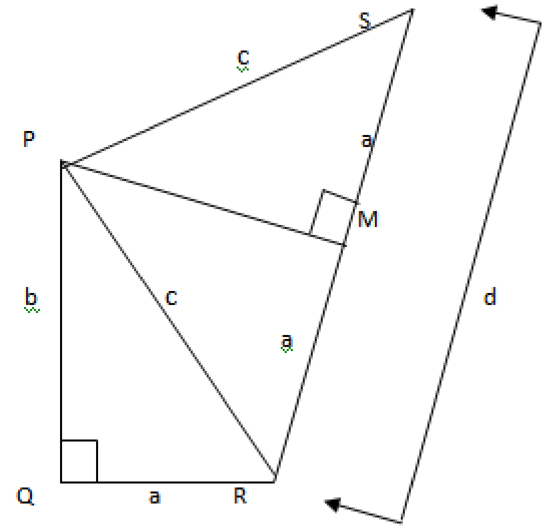

Fig No.- (8)

In □ PQRS , the lengths of all four sides are 3 cm , 4 cm , 5 cm, and 6 cm respectively.
Suppose that , the lengths of all four sides are a, b, c and d respectively.
The length of seg PS and length of seg PR are equal. Similarly, the length of seg PQ and the length seg PM are equal. The length of seg MR and the length MS are also equal.
Here taking,
l(QR) = 3 cm = a
l(PQ) = 4 cm = b
l(PS) = 5 cm = c
l(RS) = 6 cm = d
But,
l(PS) = *l*(PR) =5 cm
l(PR) = 5 cm = c
similarly,
l(QR) = *l*(MR) = *l*(MS) = 3 cm
l(MR) = *l*(MS) = 3 cm = a
*a*nd,
l(PQ) = *l*(PM) = 4 cm
l(PM) = 4 cm = b

From □ PQRS ,
Step (1)Taking first a right angled triangle△ PQR.

In △ PQR ,
l(QR) = a, *l*(PQ) = b, *l*(PR)= c
m PQR = 90^0
Side PQ and side QR are both sides forming the right angle. Seg PR is it hypotenuse.

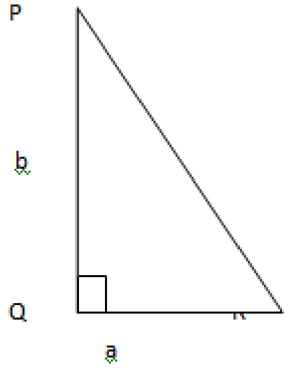

Fig No. – (9)

Area of △ PQR = A (△ PQR)
= ½x Product of the lengths of sides forming the right angle.
= ½ x One side forming the right angle x Other side forming the right angle
= ½x *l*(QR) x *l*(PQ)
= ½ x a x b
=ab/2
Area of △PQR = ab/2……………………….(1)

Step (2) Let us now taking an isosceles triangle△ PRS.

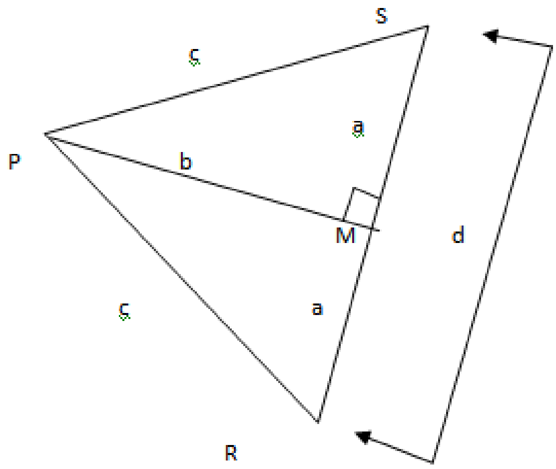

Fig No. – (10)

In △ PRS,
The length of seg PS and the length of seg PR are equal.
Similarly,
The length of seg MR and the length of seg MS are equal. PMR and △ PMS are right angled triangles. These two right angled triangles are congruent also. Seg PM is the height and side RS is the base.
Height , *l*(PM) = b
Base , *l*(RS) = d

Area of △PRS = A (△PRS)
= ½ x Base x height
= ½ x l(RS) x l(PM)
= ½ x d x b
= db/2
Area of △PRS = db/2 ------------------------(2)

From equation no. (1) and (2)
Area of □ PQRS = A (△PQR) + A (△PRS)
= ab/2 + db/2
= b/2 (a + d)
Area of □ PQRS = b/2 (a + d)

New Formula no . (1)

Area of a scalene quadrilateral = b/2 (a + d)
From an isosceles triangle △PRS,

Step (3) (A) now taking a right angled triangle△ PMR.
In△ PMR ,
Seg PM and seg MR are both sides forming the right angle. Seg PR is it's hypotenuse.
L(MR) = a, l(PM) = b l(PR) = c and m ∠PMR= 90^0

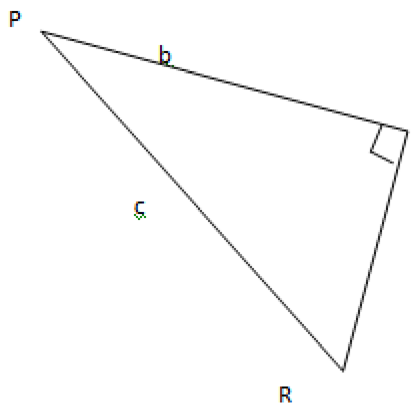

Fig. No – (11)

Area of△ PMR = A (△PMR)
= ½ x Product of the lengths of the sides forming the right angle.
= ½ x One side forming the right angle x other side forming the right angle.
= ½ x l(MR) x l(PM)
= ½ x a x b
= ab/2
Area of △PMR = ab/2 ------------------(3)

(B) Taking a right angled triangle PMS△ also

Similarly , in △ PMR**,**
Seg PM and seg MS are both sides forming the right angle. Seg PS is it's hypotenuse.
l(MS) = a, l(PM) = b, l(PS) = c
and m PMS = ∠ 90^0

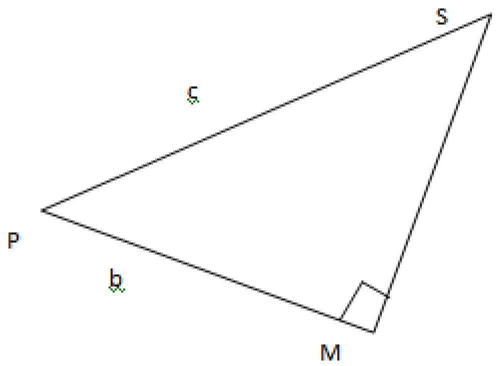

Fig No. – (12)

Area of △ PMS = A (△PMS)
= ½ x product of lengths of the sides forming the right angle.
= ½ x one side forming the right angle x other side forming the right angle.
= ½ x l(MS) x l(PM)
= ½ x a x b
= ab/2

Area of △PMS = ab/2-------------------------(4)

From equation (3) and (4)
Area of △PRS = A (△PMR) + A (△PMS)
= ab/2 + ab/2
= ab+ab/2
= 2ab/2
= ab
Area of △ PRS = ab--------------------------------(5)
From equations no. (1) and (5)
Area of □PQRS = A (△PQR) + A (△PRS)
= ab/2 + ab
= ab/2 + 2ab/2
= ab+2ab/2
= 3ab/2
= 3/2 ab
Area of □PQRS = 3/2 ab
New Formula no. (2)
Area of scalene quadrilateral = 3/2 ab
Now consider the following example :-
Ex. (1) Find the area of □ ABCD.
In □ ABCD , l(BC)= 6 cm.
L(AB) = 8 cm l(AD) = 10 cm,
l(CD) = 12 cm and m∠ABC= 90^0

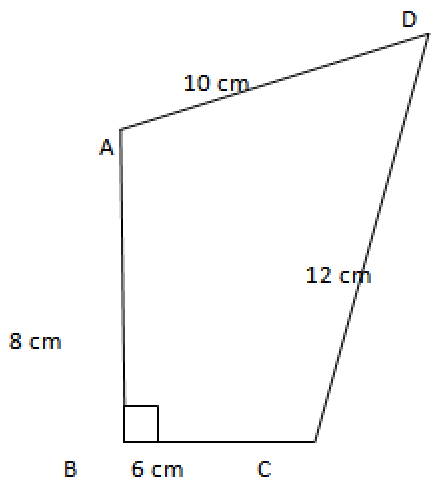

Fig No. – (13)

□ABCD is scalene quadrilateral.

In □ ABCD,

Out of the four sides , the length of first three consecutive sides (6cm, 8cm, 10cm) is a Pythagorean Triple.

And, the length of fourth side CD (12cm) is double the length of the smallest side BC (6cm)

Let, a = 6 cm , b = 8 cm , c = 10 cm and d = 12 cm

By using the new formula no (1)

Area of □ ABCD = (□ABCD)

= b/2(a + d)

= 8/2 (6 + 12)

The simplest form of 8/2 is 4

= 4 x 18= 72 sq.cm.

Area of □ ABCD = 72 sq.cm

By using the new formula no. (2)

Area of □ ABCD =A □ ABCD)

=3/2 ab

=3/2 x 6 x 8

= 3x 6 x 8/2

=3 x 48/2

=3 x 48/2

The simplest form of 48/2 is 24

= 3 x 24 = 72sq.cm

Area of □ ABCD =72 sq.cm

Verification :-

To find the area of □ABCD ,

Diagonal AC is drawn, so that this quadrilateral is divided into convenient two triangles, namely

△ABC and ACD.

1) Let us first find the area of △ABC

Area of a right angled triangle ABC

= A (△ABC)

= ½ x Product of the lengths of the sides forming the right angle.

= ½ x ab

= ½ x 6 x 8

=6 x 8/2

= 48/2

= 24 sq.cm

Area of △ ABC = 24 sq.cm

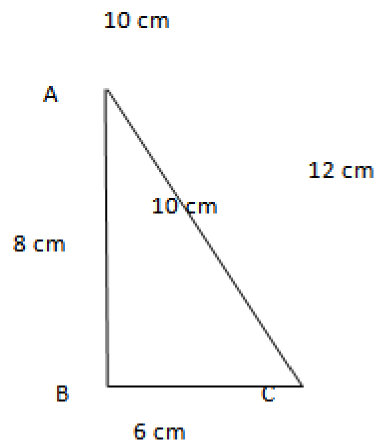

Fig No. – (14)

2) Let us now find the area of △ACD in a right angled triangle ABC,

By using Pythagoras theorem

$AC^2 = AB^2 + BC^2$

$= 8^2 + 6^2$

$= 64 + 36$

$AC^2 = 100$

$AC^2 = 10^2$

Removing the square on both sides,

AC = 10 cm

In △ACD, l(AC) = l(AD) = 10 cm and l(CD) = 12 cm

△ACD is an isosceles triangle.

By using the formula of Heron's

Perimeter of △ ACD = 10+12+10 = 32 cm.

Semiperimeter of △ ACD,s = 32/2 = 16 cm

Area of △ACD = $\sqrt{16 (16-10) (16- 12) (16 – 10)}$

$= \sqrt{16 \times 6 \times 4 \times 6}$

$= \sqrt{(16 \times 4) \times (6 \times 6)}$

$= \sqrt{64 \times 36}$

$= \sqrt{64} \times \sqrt{36}$

The square root of 64 is 8 and the square root of 36 is 6

= 8 x 6 = 48 sq.cm.

Area of △ ACD = 48 sq.cm.

Area of △ACD = 48 sq,cm.

Area of □ABCD = A (△ABC) A (△ACD)

= 24 + 48

= 72 sq.cm.

Area of □ABCD = 72 sq.cm.

V. Explanation

These above examples are solved using the new formulae No. (1) and (2) for finding the area of new quadrilateral.
From above solved example and it's verification, we note that – the value of solved example and the value of it's verification are equal.
Hence , the new formulae No (1) and (2) are proved.

VI. Conclusion

These above two new formulae are invented for finding the area of a scalene quadrilateral.

New formulae no. (1) And (2) are as follows :-
Area of □ PQRS = b/2 (a + d)

New formula no (1)
Area of a scalene quadrilateral = b/2 (a + d)
OR
Area of □ PQRS = 3/2 ab

New formula no. (2)
Area of a scalene quadrilateral = 3/2 ab

From above two new formulae, we can find out the area of a scalene quadrilateral of first new type. Out of two new formulae, any one formula can use to find the area of such quadrilateral.
These new formulae are useful in educational curriculum , building and bridge construction and department of land records.

References

[1] Geometry concept & Pythagoras theorem.